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A STABLE FRICTION COMPENSATION SCHEME FOR MOTION CONTROL SYSTEMS

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In mechanical positioning control systems, friction phenomenon makes undesirable responses. Friction compensation scheme is frequently implemented to the control system together with the friction estimator in order to cancel the effect of the friction. However, the error in the estimation of the friction results in undercompensation or overcompensation, which also makes undesirable responses. In this paper, a new friction compensation scheme is proposed. The stability analysis is performed based on the circle criterion, which provides a sufficient condition for asymptotic stability of a system. For an application example, the scheme is implemented to a mechanical positioning system of a second-order plant with nonlinear friction. The effectiveness of this scheme is illustrated with the time responses obtained by computer simulation.

Key Words: Positioning Control, Friction Compensation, Circle Criterion, Asymptotic Stability

1. INTRODUCTION

In point-to-point control for mechanical positioning system, the control objective is to achieve correct stopping at reference position within an allowed position error without any jerking motion. When high precision positioning is required, friction effect is dominant and results in undesirable responses. Friction compensation scheme is frequently used together with the friction estimator in order to cancel the effect of the friction.

Kubo, Anwar and Tomizuka (1986) has suggested a modified friction compensation scheme which adds or subtracts a constant amount to the controller output depending on the sign of the velocity when the velocity is over a threshold velocity and on the sign of the controller output when the velocity is under the threshold velocity. The modified friction compensation scheme has been implemented in position tracking control. The magnitude of the friction compensation has to be determined by trial and error in the experiment. Friction compensation by 90 percent of the Coulomb friction has been suggested because of oscillation problem. The instability is due to the inconsistence of the signs of the compensation and real friction at low velocity, which means the improperness of the suggested modified compensation scheme.

Armstrong (1988) has compensated the friction by open-loop for DC servo motor control. The friction has been modeled in consideration of the dependence of the friction upon position and velocity. The input torque is precomputed by looking up the table obtained by the experiment. Canudas and his colleagues (1986, 1989) has implemented the friction compensation scheme with the on-line friction estimation, and designed the controller under the assumption of perfect friction compensation. The schemes mentioned above adopt their own model and have the modeling error. Moreover, if the estimation speed is slower than the change rate of friction in the on-line friction estimation, there occurs some error. These errors in the estimation of friction result in undercompensation or overcompensation, which prevents the control objective.

In Chapter 2 of this paper, the steady state error due to the undercompensation is discussed and the fact that overcompensation makes a limit cycle near the reference position is explained using input-output relationship between the controller output and the net input to the system. In Chapter 3, a new friction compensation scheme is proposed, and the stability analysis is performed based on the circle criterion, which provid es a sufficient condition for asymptotic stability of a system. In Chapter 4, an application example is illustrated with the results from computer simulation.

2. CONVETIONAL POSITIONING CONTROLLER

In general, mechanical positioning systems are modeled using the inertia, J and the viscous damping coefficient. B. The block diagram of the linear positioning control system is shown in Fig. 1. Based on the linear control theory (Ogata, 1970), it is possible to achieve zero steady state position error to a step position reference by implementing the proportional (P-) controller or the proportional plus derivative (PD-) controller. In physical mechanical systems, however, there exists nonlinear friction. The friction phenomenon is complicated since the friction is subject to the conditions of surface roughness and lubrication at the contact point. The friction is not only position-dependent but also time-varing. The maximum friction force in stick state is called static friction force (stiction) and the friction force in sliding notion is called dynamic friction force. The typical friction model frequently used is shown in Fig. 2. Dynamic friction force decreases as velocity increases from 0 and approaches to a limit, which will be called Coulomb friction force, henceforth. Generally,

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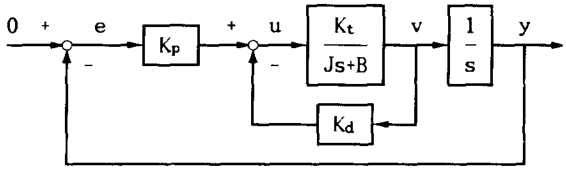


Fig. 1 The block diagram of a linear positioning control system

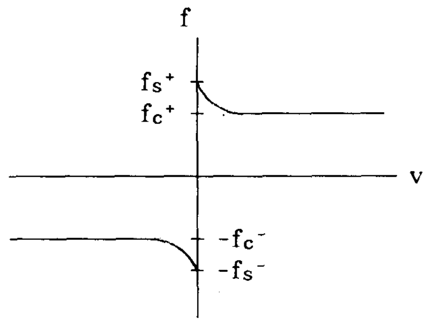


Fig. 2 The friction model of a mechanical positioning system

the friction characteristics in the positive and the negative directions are different. In Fig. 2, f_s^+ and f_s^- are the magnitudes of the static friction forces in the positive and the negative directions, respectively, and f_c^+ and f_c^- are the magnitudes of the Coulomb friction forces in the positive and the negative directions, respectiviely.

A conventional friction compensation scheme is to add or subtract the estimated friction force depending on the sign of the velocity. Fig. 3 shows the block diagram of the conventional friction compensation control system. The linear controller is tuned under the assumption of correct friction compensation. The compensated controller output, u_c is calculated by

$$u_c = u + \hat{f} \cdot \mathrm{sgn}(v) \qquad (1)$$

where u is the PD-controller coutput, $\hat{f}$ is the estimated friction, v is the velocity, and sgn(·) is the sign function.

If the friction force estimated by off-line is used in compensation, correct compensation is not possible because the friction is not only velocity and poosition-dependent but also time-varing. Even though the friction force is estimated by on-line, the estimation error is unavoidable because of the error in the friction model, the measurement error and the finite convergence rate. Especially, at low velocity the slope of the friction with respect to velocity is steep. Right before stopping or right after stiction break, the estimate can not catch up the real friction if the estimation speed is slower than the change rate of friction. Thus, overcompensation or undercompensation happens.

Undercompensation produces unsatisfactory response in point-to-point position control. When P-or PD-controller is used, there exists nonzero steady state position error to a step position reference(Yang and Tomizuka, 1988). The maximum steady state position error is

$$\max(|e_{ss}|) = \frac{f_s - f_{comp}}{K_p} \qquad (2)$$

where f_s and f_{comp} are the magnitudes of the static friction force and the friction compensation amount, respectively, and K_p is the proportional gain in the position feedback loop.

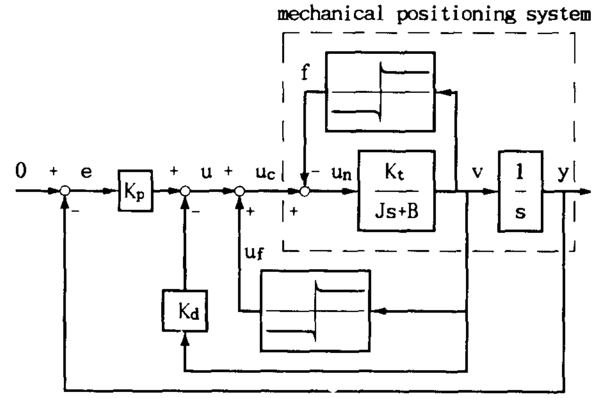


Fig. 3 The block diagram of a conventional friction compensation control system

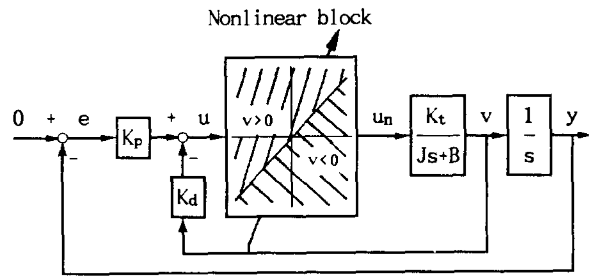


Fig. 4 The reconfigured block diagram of the conventional friction compensotion control system in the case of friction overcompensation

It has been found experimentally by Kubo(1986) that overcompensation causes oscillation near the reference position. It can be explained by figuring out the input-output relationship between the controller output and the net input to the positioning system (with both friction compensation and real friction counted). The block diagram of the system in Fig. 3 is reconfigured for stability analysis as shown in Fig. 4, which is exactly the same as that of Fig. 3 in case of friction overcompensation. The nonlinear block comprises the friction of the mechanical positioning system and compensation of the friction. If the friction is estimated correctly, the gain of the nonlinear block is exactly 1. In the case of friction overcompensation, however, the input-output relationship of the nonlinear block is represented by the two regions depending on the sign of the velocity.

For example, when the controller output, u is negative and the net input, u_n is positivie, the gain of the nonlinear block can be negative because of the overcompensation and the closed-loop system behaves as a positive feedback system, which is unstable. Then, the controller output signal diverges until the net input becomes negative, that is, the gain of the nonlinear block becomes negative and the system becomes stable. When the system is under the stable condition, the magnitude of the controller output signal decreases. These two stable and unstable conditions occur alternately, and finally will make a limit cycle. The size of the limit cycle can be obtained by the describing function of the nonlinear block. It is expected that the size depends on the amount of the overcompensation. In this study, the describing function analysis is omitted.

3. FRICTION COMPENSATION

When the conventional friction compensation scheme referred in Chapter 2 is implemented to a physical system, the instability of the system is attributed to the negative gain between u and u_n. In this chapter, a new friction compensation scheme which never makes the gain negative is proposed and the asymptotic stability of the system is proved by the circle criterion.

3.1 A New Friction Compensation Scheme

Fig. 5 shows the input-output relationships of the nonlinear block of Fig. 4 in cases of undercompensation and overcompensation for $v>0$ and $v<0$. In order for the nonlinear block to take positive gain, the relationships must be confined in the first and the third quadrants. For example, when $v>0$, the friction must be overcompensated for $u>0$, and undercompensated for $u<0$. When $v<0$, the reverse holds true. When $v=0$ and $u\neq0$, the controlling input must be overcompensated to exceed the static friction force. When $v\neq0$ and $u=0$, either undercompensation or overcompensation will do, but undercompensation is preferred not to allow the velocity overshoot. The friction compensation law is described in Table 1.

Fig. 6 shows the block diagram of a position control system with the friction compensation scheme suggested above. The friction is overcompensated or under compensated depending on the controller output signal as well as the velocity. The compensated controller output, u_c is calculated by

$$u_c=u+u_f(u,v) \tag{3}$$

where u_f is the compensation value for the friction force, which is a nonlinear function given by

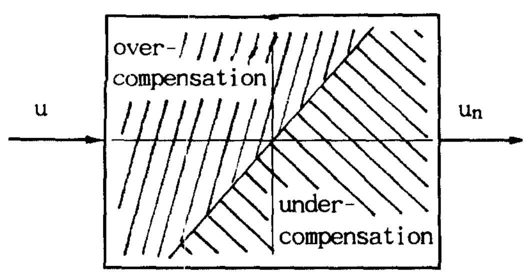
$$u_f(u,v)=\begin{cases} f_o^+ & \text{if } v\geq0 \text{ and } u>0 \\ f_u^+ & \text{if } v>0 \text{ and } u\leq0 \\ 0 & \text{if } v=0 \text{ and } u=0 \\ -f_u^- & \text{if } v<0 \text{ and } u\geq0 \\ -f_o^- & \text{if } v\leq0 \text{ and } u<0 \end{cases} \tag{4}$$

f_o and f_u are the positive values for overcompensation and undercompensation, respectively, and the superscripts '+' and '−' mean the positive and the negative directions of motion. f_o^+ and f_o^- must be big enough not to undercompensate the static friction in each direction, and f_u^+ and f_u^- must small enough not to overcompensate the Coulomb friction in each direction. For simplicity, one common overcompensation value, f_o can be used instead of f_o^+ and f_o^-, and, one common undercompensation value, f_u, instead of f_u^+ and f_u^-. Throughout this paper it is assumed that

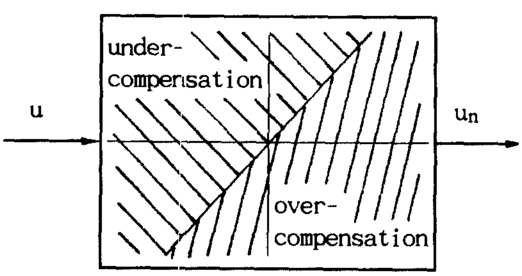
$$f_o\geq\max(f_s^+,\ f_s^-) \tag{5}$$

and

$$f_u\leq\min(f_c^+,\ f_c^-) \tag{6}$$



(a) when $v>0$



(b) when $v<0$

Fig. 5 The input-output relationships of the nonlinear block

Table 1 The friction compensation law.

v \ u	$u<0$	$u=0$	$u>0$
$v>0$	UC	UC(or OC)	OC
$v=0$	OC	0	OC
$v<0$	OC	UC(or OC)	UC

UC : Undercompensation
OC : Overcompensation

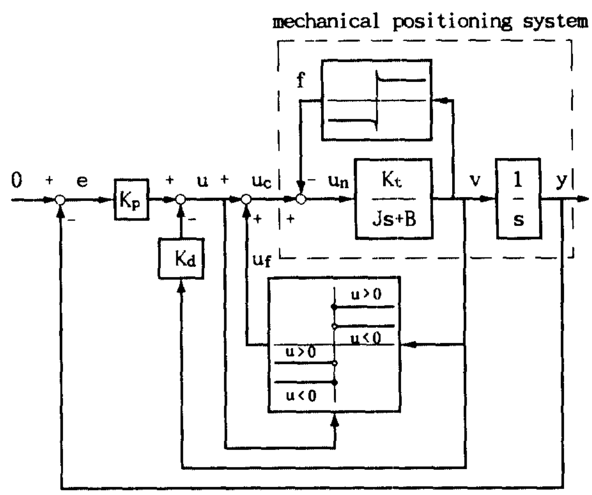


Fig. 6 The block diagram of the control system with the new friction compensation scheme

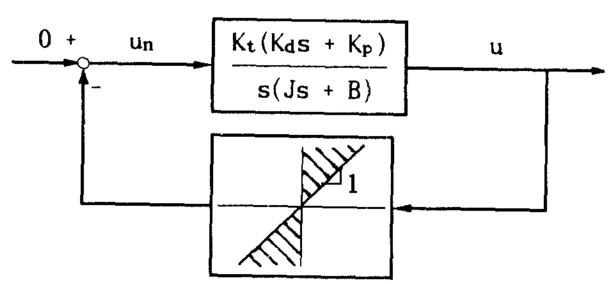


Fig. 7 The reconfigured block diagram of the control system with the new friction compention scheme

3.2 Stability Analysis

The block diagram of the system in Fig. 6 is reconfigured for stability anlysis as in Chapter 2. Fig. 7 shows the reconfigured block diagram of the control system with the new friction compensation scheme. The system in Fig. 7 is equivalent to that of Fig. 6 with respect to stability. The input-output relationship of the nonlinear block is represented by a sector bounded by two lines with slopes of 1 and infinity.

Based on the circle criterion, the nonlinear feedback system is asymptotically stable, if the frequency response of $G(s)$, the transfer function of the linear block in Fig. 7, is outside of the disk in Fig. 8 and does not encircle the disk clockwisely. Since this system is a secondorder system, the phase of the frequency response takes a value between 0 and $-\pi$. If

$$\frac{JK_p - BK_d}{K_t K_d^2} \leq 1 \quad (7)$$

the frequency response curve does not cross the disk, which is proved in appendix. Therefore, if Eq. (7) is satisfied, the frequency response curve does not cross and not encircle the disk of Fig. 8, that is, the nonlinear system is asymptotically stable. The above condition is a sufficient condition for asymptotic stability.

3.3 Extension to Tracking Control

In the previous section, the stability analysis has been applied to the PD-control system with the friction compensation scheme for regulation. In this section, it is proved that both the regulation and the tracking PD-control systems are equivalent with respect to the stability under some conditions.

Consider a linear plant,

$$\dot{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u_n \quad (8)$$

and the output,

$$Y = \mathbf{C}\mathbf{x} \quad (9)$$

where u_n is the net input to the linear plant, which is a nonlinear function of u and v as described in Fig. 6.

$$u_n = \phi(u, v) = \phi(u, \dot{y}) \quad (10)$$

For the PD-controller in Fig. 6, the controlling input, u is

$$u = K_p e - K_d \dot{y} \quad (11)$$

where

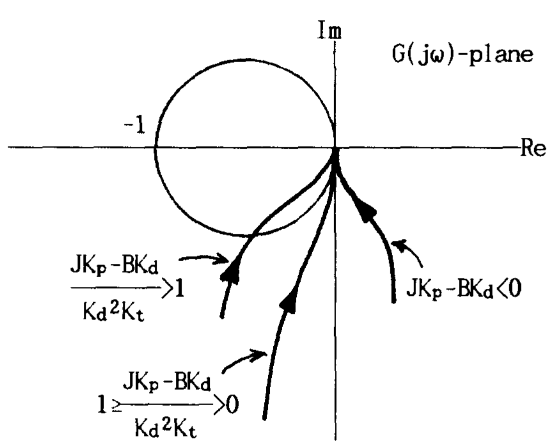


Fig. 8 The nyquist plot of $G(s)$ and the disk of circle criterion for the nonlinear block

$$e = y_r - y \quad (12)$$

For regulation, $y_r = 0$, and Eq. (11) becomes

$$u = -K_p y - K_d \dot{y} \quad (13)$$

The stability of the system given by Eqs. (8) ~ (10) and (13) (the PD-control system with the friction compensation scheme for regulation) has been analyzed in Section 3.2.

For tracking with the constant command, y_r, a deviation variable, y' is defined as

$$y' = y - y_r \quad (14)$$

If we can find $\mathbf{x}_r$ such that

$$\mathbf{C}\mathbf{x}_r = y_r \quad (15)$$

and

$$\mathbf{A}\mathbf{x}_r = 0 \quad (16)$$

then, by letting

$$\mathbf{x}' = \mathbf{x} - \mathbf{x}_r \quad (17)$$

the Eqs. (8), (9) and (10) become

$$\dot{\mathbf{x}}' = \mathbf{A}\mathbf{x}' + \mathbf{B}u_n \quad (18)$$

$$y' = \mathbf{C}\mathbf{x}' \quad (19)$$

and

$$u_n = \phi(u, \dot{y}') \quad (20)$$

respecitively, and Eq. (11) becomes

$$u = -K_p y' - K_d \dot{y}' \quad (21)$$

Eqs. (18), (19) and (20) are identical to Eqs. (8), (9) and (10), and Eq. (21) is identical to the regulation PD-control law of Eq. (13). Therefore, both the regulation and the tracking PD-control systems are equivalent with respect to the stability, if there exists any $\mathbf{x}_r$ satisfying Eqs. (15) and (16).

4. SIMULATIONS

In computer simulation, a second crder plant with the PD-controller for constant command tracking is adopted in order to illustrate and compare the time response of the system with the suggested friction compensation to that with the conventional one. The second order plant in Fig. 1 can be represented as Eqs. (8) and (9), were the parameters are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -B/J \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ K_t/J \end{bmatrix}, \quad \mathbf{C} = [1 \ 0] \quad (22)$$

If we set $\mathbf{x}_r$ as

$$\mathbf{x}_r = [y_r \ 0]^T \quad (23)$$

then, Eqs. (15) and (16) are satisfied. Therefore, based on Section 3.3, this constant command tracking control system has the same stability property as the regulation control system.

In simulation, $K_t = 1$, $J = 2.0$ kg, $B = 1.0$ kg/s, $f_c = 10$ kgm/s², $f_s = 13$ kgm/s², where the same friction characteristics in the positive and the negative directions was assumed. These values are close to a real laboratory XY-motion sys-

Table 2 Various cases of friction compensation

	Compensation	f_u	f_o
(1)	Exact compensation	—	—
(2)	Overcompensation	14	14
(3)	Undercompensation	9	9
(4)	Proposed Compensation	5	19.5

(Unit : kgm/s²)

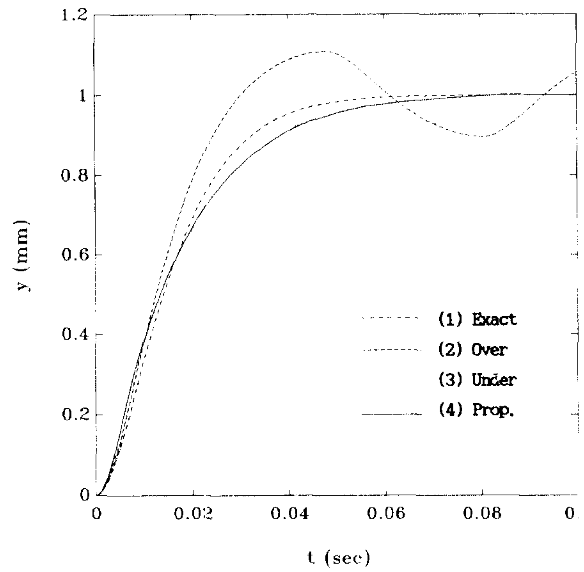


Fig. 9 The time responses of y for various cases of friction compensationr

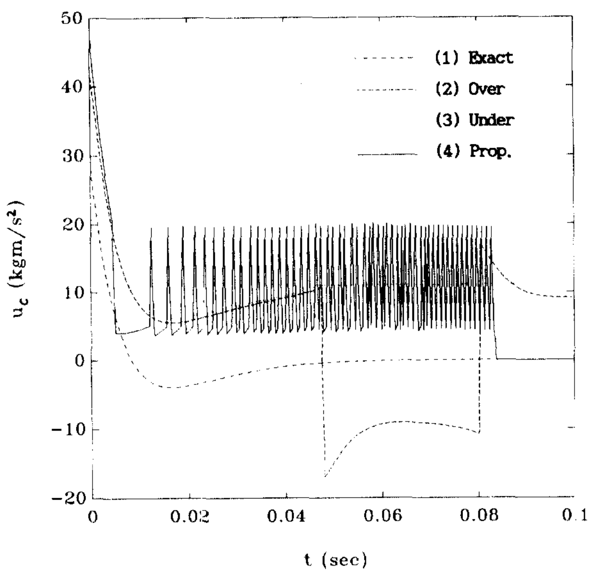


Fig. 10 The time responses of the compesnated controlling input, u_c, for various cases of friction compensation

tem. The controller parameters are set as $K_p = 2.825 \times 10^4$, and $K_d = 474$, which satisfy the stability condition of Eq. (7). These values are selected arbitrarily just to illustrate the responses of the control system. Computer simulations were performed for 4 cases of friction compensation schemes listed in Table 2.

Figs. 9 and 10 show the time responses of y and the compensated controlling input, u_c, respectively. As expected, the friction overcompensation (Case (2)) causes overshoot and a limit cycle, and the friction undercompensation (Case (3)) results in the steady state error. The response of Case (4) shows neither overshoot nor steady state error, and is close to that of exact friction compensation.

5. CONCLUSIONS

In point-to-point position control, friction causes unsatisfactory response. Generally, friction compensation scheme is implemented to the control system together with the friction estimator in order to cancel the effect of the friction. However, the error in the on-line estimation of the friction results in undercompensation or overcompensation. The fact that overcompensation makes a limit cycle near the reference position has been explained using input-outut relationship between the controller output and the net input to the system.

A new friction compensation scheme has been proposed. It has been shown that the stability analysis can be performed based on the circle criterion, which provides a sufficient condition for asymptotic stability. The stability of the system has been proven not only in the case of the regulation control but also in the case of the constant command tracking control under some conditions. For an application example, the proposed scheme has been implemented to a mechanical positioning system of a second-order plant with nonlinear friction. The effctiveness of the scheme has been illustrated with the time responses obtained by computer simulation. The experimental verification of the scheme is left.

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APPENDIX

The frequency response of the transfer function of the linear block in Fig. 7, $G(j\omega)$ is given by

$$\begin{aligned} G(j\omega) &= \frac{K_t(jK_d\omega + K_p)}{j\omega(jJ\omega + B)} \\ &= \frac{K_t(BK_d - JK_p)}{J^2\omega^2 + B^2} + j\frac{K_t(JK_d\omega^2 + BK_p)}{\omega(J^2\omega^2 + B^2)} \\ &= \mathrm{Re} + j\mathrm{Im}. \end{aligned} \tag{24}$$

The equation of the disk in Fig. 8 is represented by

$$\left(\mathrm{Re}+\frac{1}{2}\right)^2+\mathrm{Im}^2=\frac{1}{4} \tag{25}$$

From Eqs. (24) and (25), a quadratic equation for ω^2 is obtained as

$$\frac{BK_p^2}{K_d^2}+\left[\frac{J^2K_p^2}{K_d^2}+\left(1+\frac{BK_d-JK_p}{K_d^2K_t}\right)B_2\right]\omega^2 + \left[1+\frac{BK_d-JK_p}{K_d^2K_t}\right]J^2\omega^4=0 \tag{26}$$

It can be easily shown that if

$$1+\frac{BK_d-JK_p}{K_d^2K_t}<0 \tag{27}$$

Eq. (26) has one real positive solution for ω^2. Otherwise, it does not have any real positive solution for ω^2. Therefore, under the condition given by Eq. (7) the frequency response curve does not cross the disk.